Key 26
— 4th and 5th Bulleted Exercise + Bonus

- 4th Bulleted Exercise

Determine the mean and standard deviation of the number thrown by a six-sided die (equal probability on each one through six)

sides of a die	0	1	2	3	4	5	6
probability	.12	.15	.18	.26	.10	.08	.06

$\mu = \sum (x)\, P(x) = 0(.12)+1(.15)+2(.18)+3(.26)+4(.10)+5(.08)+6(.06)$

mean or expected value — random value — probability variable

$= 0 + .15 + .36 + .78 + .40 + .40 + .36$

$= 2.45$

— answer: $\mu = 2.45$

In order to find the standard deviation of the six sided die first find the variance

$\sigma^2 = \sum (x-\mu)^2 P(x) = \sum x^2 P(x) - \mu^2$

$\sigma^2 = 0(.12)+1(.15)+4(.18)+9(.26)+16(.10)+25(.08)+36(.06) - (2.45)^2$

$= 8.97 - (2.45)^2$

$\sigma^2 = 2.9675$

$\sigma = \sqrt{\sigma^2}$

$\sigma = \sqrt{2.9675}$

— answer: $\sigma = 1.72264$

- 5th Bulleted Exercise

Calculate the mean and standard deviation of the number of heads thrown in 100 tosses of a coin (and also for the number of heads thrown in 10,000 tosses of a coin). Additionally (simulation), toss a die 100 times and calculate the average of 100 numbers thrown uppermost. Does this sample average come close to the mean you calculated from the probability model.

100 tosses of a coin

$\mu = np$

mean

probability of success

# of trials

so: $\mu = 100(.50)$

- answer: $\mu = 50$

$\sigma = \sqrt{np(1-p)}$

1 - probability of success

standard deviation

probability of success

number of trials

so: $\sigma = \sqrt{100(.50)(1-.50}$

$\sigma = \sqrt{100(.50)(.50)}$

$\sigma = \sqrt{25}$

- answer: $\sigma = 5$

10000 tosses of a coin

$\mu = 10000(.50)$

- answer: $\mu = 5000$

$\sigma = \sqrt{10000(.50)(.50)}$

$= \sqrt{2500}$

- answer: $\sigma = 50$

Simulation

I use RandInt on my calculator to obtain the avg of the random #'s on a six-side die rolled 100 times so on my calculator I typed (1, 6, 100) which

represented: (1, 6, 100).
range I wanted to go from i.e. six sides on the die
how many times I wanted the random #'s to occur

So now that I had all of my random numbers I found the average of the 100 numbers thrown uppermost. I came up with 3.5. Which is exactly the mean from the probability model.

* Bonus problems

1) Determine: a) μ b) σ^2 c) σ d) $\sqrt{(\mu x^2 - \mu^2_x)}$
e) $S = \sqrt{\frac{\sum(x-\mu)^2}{n-1}} = \sqrt{\frac{n}{n-1}}\,\sigma$

for $\{2.3, 5.4, 7.9, 6.6, 2.1\}$

a) $\bar{x} = 4.86$ (calculator answer)

$\mu = \frac{2.3 + 5.4 + 7.9 + 6.6 + 2.1}{5} = \frac{24.3}{5} = 4.86$

b) $\sigma_x^2 = 5.3464$ (calculator answer)

$\sigma^2 = \frac{\sum(x - \mu_x)^2}{n}$

$\sigma^2 = \frac{(2.3-4.86)^2 + (5.4-4.86)^2 + (7.9-4.86)^2 + (6.6-4.86)^2 + (2.1-4.86)^2}{5}$

$\sigma^2 = \frac{(-2.56)^2 + (.54)^2 + (3.04)^2 + (1.74)^2 + (-2.76)^2}{5}$

$\sigma^2 = \frac{6.5536 + .2916 + 9.2416 + 3.0276 + 7.6176}{5}$

$\sigma^2 = 5.3464$

c) $\sigma_x = 2.312228362$ (calculator answer)

$\sigma = \sqrt{\sigma^2}$

$\sigma = \sqrt{5.3464}$

$\sigma = 2.312228362$

d) $\sqrt{(\mu_x^2 - \mu_x^2)}$

$= \sqrt{\frac{(2.3)^2 + (5.4)^2 + (7.9)^2 + (6.6)^2 + (2.1)^2}{5} - (4.86)^2}$

$= \sqrt{\frac{144.83}{5} - 23.6196}$

$= \sqrt{28.966 - 23.6196}$

$= \sqrt{5.3464}$

$= 2.312228362$

e) $S = \sqrt{\frac{\sum (x-\mu)^2}{n-1}} = \sqrt{\frac{n}{n-1}}\,\sigma$

$S_x = 2.58514899$ (calculator answer)

$S = \sqrt{\frac{(2.3-4.86)^2 + (5.4-4.86)^2 + (7.9-4.86)^2 + (6.6-4.86)^2 + (2.1-4.86)^2}{5-1}} = \sqrt{\frac{n}{n-1}}\,\sigma$

$S = \sqrt{\frac{(-2.56)^2 + (.54)^2 + (3.04)^2 + (1.74)^2 + (-2.76)^2}{4}} = \sqrt{\frac{5}{5-1}}\,(2.312228362)$

$S = \sqrt{\frac{6.5536 + .2916 + 9.2416 + 3.0276 + 7.6176}{4}} = \sqrt{\frac{5}{4}}\,(2.312228362)$

$S = 2.585149899 = 2.585149899$

2) no question

3) no question

4) Refer to problem 2. For any fixed large # the total casino winnings up to that point in time approx follow a normal (i.e. "bell") curve w/

bell curve mean $= \mu \#$ bell curve standard deviation $= \sigma\sqrt{\#}$

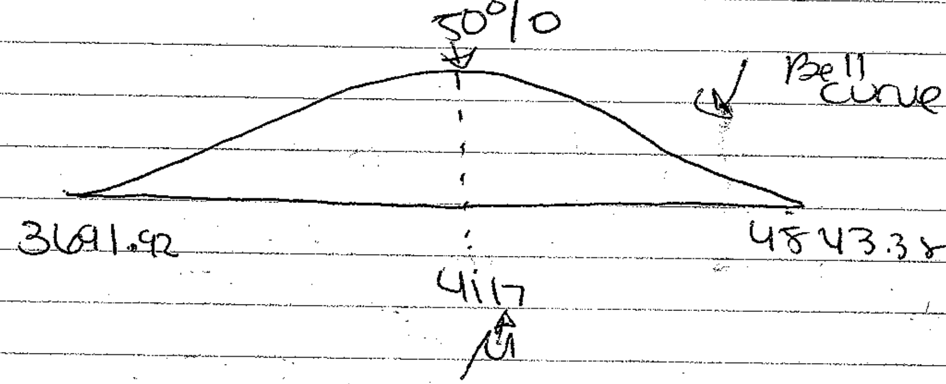
a) Evaluate 68% interval give

$.441765(10\,000) + 4.2573\sqrt{10000} = 4543.38$

~~[illegible]~~ $.441765(10000) - 4.2573\sqrt{10000} = 3691.92$

b) What is the chance the casino earns a total of less than $.411765(10\,000)$. after 10 000 plays?

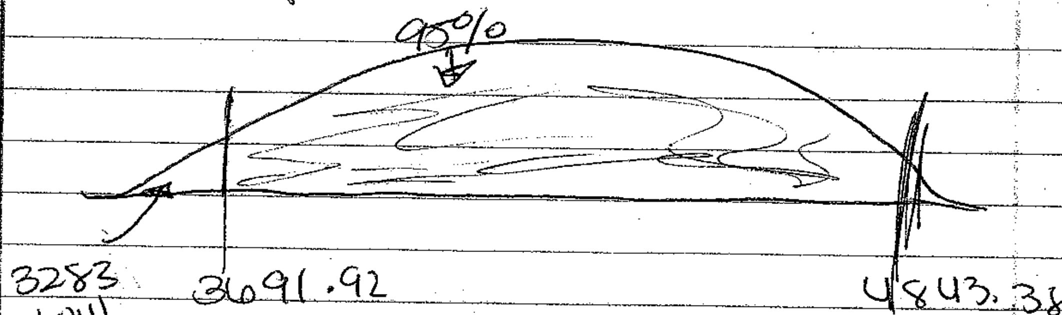
~ 4117



c) What is the chance the casino earns a total of less than

$.411765(10\,000) - 1.96(4.2573)\sqrt{10000}$

after a 10000 plays? $= \sim 3283$



will placed here

at

.25%